

	Discrete	Continuous
Support	Countable set of values	Uncountable set of values
Probabilities	pmf: $P(X = x) = p(x)$	$P(X = x) = 0 \neq f(x)$ for all x $P(a \leq X \leq b) = \int_a^b f(x)dx = F(b) - F(a)$
Expected Value	$\mu_X = \mathbb{E}[X] = \sum_x xp(x)$ $\mathbb{E}[g(X)] = \sum_x g(x)p(x)$	$\mu_X = \mathbb{E}[X] = \int_{-\infty}^{\infty} xf(x) dx$ $\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x)f(x) dx$
Independence	$p_{XY}(x, y) = p_X(x)p_Y(y)$	$f_{XY}(x, y) = f_X(x)f_Y(y)$

Table 1: Differences between Discrete and Continuous Random Variables

Probability Mass Function $p(x)$	Probability Density Function $f(x)$
Discrete Random Variables	Continuous Random Variables
$p(x) = P(X = x)$	$f(x) \neq P(X = x) = 0$
$p(x) \geq 0$	$f(x) \geq 0$
$p(x) \leq 1$	$f(x)$ can be greater than one!
$\sum_x p(x) = 1$	$\int_{-\infty}^{\infty} f(x) dx = 1$
$F(x_0) = \sum_{x \leq x_0} p(x)$	$F(x_0) = \int_{-\infty}^{x_0} f(x) dx$

Table 2: Properties of probability mass function (pmf) versus probability density function.

Definition of R.V.	$X: S \rightarrow \mathbb{R}$ (RV is a fixed function from sample space to reals)
Support	Set of all values the RV can take
CDF	$F(x_0) = P(X \leq x_0)$
Definition of Variance	$\sigma_X^2 = Var(X) = E[(X - E[X])^2]$
Shortcut for Variance	$Var(X) = E[X^2] - (E[X])^2$
Definition of Std. Dev.	$\sigma_X = \sqrt{\sigma_X^2}$
Covariance	$\sigma_{XY} = Cov(X, Y) = E[(X - E[X])(Y - E[Y])]$
Cov. and Independence	X, Y indep. $\Rightarrow Cov(X, Y) = 0$ but $Cov(X, Y) = 0 \not\Rightarrow X, Y$ indep.
Functions and Independence	X, Y indep. $\Rightarrow g(X), h(Y)$ indep. where g, h are any functions
Shortcut for Covariance	$Cov(X, Y) = E[XY] - E[X]E[Y]$
Definition of Correlation	$\rho_{XY} = Corr(X, Y) = \sigma_{XY} / (\sigma_X \sigma_Y)$
Expectations of Functions	$E[g(X)] \neq g(E[X])$
Linear Functions	<p>$E[a + bX] = a + bE[X]$ where a, b are constants and X is a RV</p> <p>$Var(a + bX) = b^2 Var(X)$ where a, b are constants and X is a RV</p> <p>$E[X_1 + \dots + X_k] = E[X_1] + \dots + E[X_k]$ where X_1, \dots, X_k are any RVs</p> <p>$Var(X_1 + \dots + X_k) = Var(X_1) + \dots + Var(X_k)$ if X_1, \dots, X_k are independent RVs</p> <p>$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$ for any RVs X, Y and constants a, b</p>

Table 3: Essential facts that hold for *all* random variables, continuous or discrete

	Sample Statistic	Population Parameter	Population Parameter
Setup	Sample from a population	Population viewed as list of objects	Population viewed as a RV
Mean	$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$	$\mu_X = \frac{1}{N} \sum_{i=1}^N x_i$	Discrete $\mu_X = \sum_x xp(x)$ Continuous $\mu_X = \int_{-\infty}^{\infty} xf(x) dx$
Variance	$s_X^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$	$\sigma_X^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_X)^2$	$\sigma_X^2 = E[(X - E[X])^2]$
Std. Dev.	$s_X = \sqrt{s_X^2}$	$\sigma_X = \sqrt{\sigma_X^2}$	$\sigma_X = \sqrt{\sigma_X^2}$
Covariance	$s_{XY} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$	$\sigma_{XY} = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_X)(y_i - \mu_Y)$	$\sigma_{XY} = E[(X - \mu_X)(Y - \mu_Y)]$
Correlation	$r_{XY} = s_{XY}/(s_X s_Y)$	$\rho_{XY} = \sigma_{XY}/(\sigma_X \sigma_Y)$	$\rho_{XY} = \sigma_{XY}/(\sigma_X \sigma_Y)$