

Homework questions, week 3

Econ 103

1 Daily Homework questions

The questions in bold font are due on **Thursday 9th June**. You do not need to hand in the questions that are not in bold, though these will be useful to complete for your own understanding.

Lecture 8 - Discrete Random Variables II

Textbook questions:

Chapter 4: 7, **15**, 27, **29**

Additional questions:

1. **Fill in the missing details from class to calculate the variance of a Bernoulli Random Variable *directly*, that is *without* using the shortcut formula. (Lecture 8, slide 11)**
2. Prove that the Bernoulli Random Variable is a special case of the Binomial Random variable for which $n = 1$. (Hint: compare pmfs.)

Lecture 9 - Discrete Random Variables III

Textbook questions:

Chapter 5: 1, 3, **5**, **11**, 13

Additional questions:

1. **Suppose that X is a random variable with support $\{1, 2\}$ and Y is a random variable with support $\{0, 1\}$ where X and Y have the following joint distribution:**

$$\begin{aligned} p_{XY}(1, 0) &= 0.20, & p_{XY}(1, 1) &= 0.30 \\ p_{XY}(2, 0) &= 0.25, & p_{XY}(2, 1) &= 0.25 \end{aligned}$$

- (a) Express the joint distribution in a 2×2 table.
 - (b) Using the table, calculate the marginal probability distributions of X and Y .
 - (c) Calculate the conditional probability distribution of $Y|X = 1$ and $Y|X = 2$.
 - (d) Calculate $E[Y|X]$.
 - (e) What is $E[E[Y|X]]$?
 - (f) Calculate the covariance between X and Y using the shortcut formula.
2. Fill in the missing steps from lecture to prove the shortcut formula for covariance:

$$Cov(X, Y) = E[XY] - E[X]E[Y]$$

3. (HARD) Let X_1 be a random variable denoting the returns of stock 1, and X_2 be a random variable denoting the returns of stock 2. Accordingly let $\mu_1 = E[X_1]$, $\mu_2 = E[X_2]$, $\sigma_1^2 = Var(X_1)$, $\sigma_2^2 = Var(X_2)$ and $\rho = Corr(X_1, X_2)$. A *portfolio*, Π , is a linear combination of X_1 and X_2 with weights that sum to one, that is $\Pi(\omega) = \omega X_1 + (1 - \omega)X_2$, indicating the proportions of stock 1 and stock 2 that an investor holds. In this example, we require $\omega \in [0, 1]$, so that *negative* weights are not allowed. (This rules out short-selling.)

- (a) Calculate $E[\Pi(\omega)]$ in terms of ω , μ_1 and μ_2 .
- (b) If $\omega \in [0, 1]$ is it possible to have $E[\Pi(\omega)] > \mu_1$ and $E[\Pi(\omega)] > \mu_2$? What about $E[\Pi(\omega)] < \mu_1$ and $E[\Pi(\omega)] < \mu_2$? Explain.
- (c) Express $Cov(X_1, X_2)$ in terms of ρ and σ_1, σ_2 .
- (d) What is $Var[\Pi(\omega)]$? (Your answer should be in terms of ρ, σ_1^2 and σ_2^2 .)
- (e) Using part (d) show that the value of ω that minimizes $Var[\Pi(\omega)]$ is

$$\omega^* = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

In other words, $\Pi(\omega^*)$ is the *minimum variance portfolio*.

- (f) If you want a challenge, check the second order condition from part (e).

Lecture 10 - Continuous Random Variables I

Textbook questions: none

Additional questions:

1. Suppose that X is a random variable with the following PDF

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Graph the PDF of X .
 - (b) Show that $\int_{-\infty}^{\infty} f(x) dx = 1$.
 - (c) What is $P(0.5 < X < 1.5)$?
2. **A random variable is said to follow a Uniform(a, b) distribution if it is equally likely to take on any value in the range $[a, b]$ and never takes a value outside this range. Suppose that X is such a random variable, i.e. $X \sim \text{Uniform}(a, b)$.**
- (a) What is the support of X ?
 - (b) Explain why the PDF of X is $f(x) = 1/(b - a)$ for $a \leq x \leq b$, zero elsewhere.
 - (c) Using the PDF from part (b), calculate the CDF of X .
 - (d) Verify that $f(x) = F'(x)$ for the present example.
 - (e) Calculate $E[X]$.
 - (f) Calculate $E[X^2]$. *Hint:* recall that $b^3 - a^3$ can be factorized as $(b - a)(b^2 + a^2 + ab)$.
 - (g) Using the shortcut formula and parts (e) and (f), show that $\text{Var}(X) = (b - a)^2/12$.

Lecture 11 - Continuous Random Variables II

Textbook questions: Chapter 4: 19, **21, 23** (*When necessary, use R rather than the Normal tables in the front of the textbook.*)

1. **Suppose that $X \sim N(0, 16)$ independent of $Y \sim N(2, 4)$. Recall that our convention is to express the normal distribution in terms of its mean and variance, i.e. $N(\mu, \sigma^2)$. Hence, X has a mean of zero and variance of 16, while Y has a mean of 2 and a variance of 4. In completing some parts of this question you will need to use the R function `pnorm` described in class. In this case, please write down the command you used as well as the numeric result.**
- (a) Calculate $P(-8 \leq X \leq 8)$.
 - (b) Calculate $P(0 \leq Y \leq 4)$.
 - (c) Calculate $P(-1 \leq Y \leq 6)$.
 - (d) Calculate $P(X \geq 10)$.

2 R Tutorials

You should complete R Tutorial #3 by **Thursday 9th June**.

R tutorials will be posted on Piazza, with solution code.